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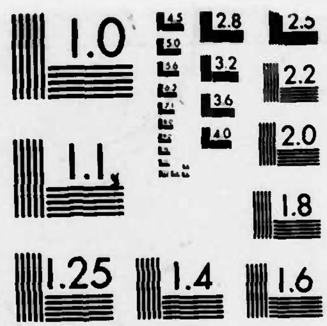

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ABSTRACT

This interim report describes the research carried out by Professors Sanjoy K. Mitter and Bernard Levy and Mr. Yehuda Avniel and Mr. Saul Gelfand during the time period March 15, 1983 to March 14, 1984, with support extended by the Air Force Office of Scientific Research under Grant AF-AFOSR 82-0135A.

The principal investigator was Professor Sanjoy Mitter. The contract monitor was Dr. J. Burns of the AFOSR Directorate of Mathematical and Information Sciences.

Research was carried out on the following main topics:

1. Linear and Nonlinear Filtering and related Scattering and Inverse Scattering Problems, and
2. Stochastic Control with Partial Observations.

Technical details of the research may be found in the reports, theses, and papers cited in the references. A list of publications supported wholly or partially by this grant is included at the end of this report.

1. Introduction

Two fundamental aspects of stochastic system theory are filtering and stochastic control theory. Indeed, the two aspects are interrelated in the sense that stochastic control in the presence of incomplete and uncertain information about the state of the system. This is the domain of filtering theory. Moreover, the problem of parameter identification could be considered as a special case of nonlinear filtering and the problem of optimal adaptive control (suitably formulated) can be considered as a special case of stochastic control with partial uncertain observations. One must remark that this necessitates taking a Bayesian point of view.

Stochastic Calculus of Variations could be considered as a special case of Stochastic Control. This is well known in the deterministic situation and indeed the two fields could be considered equivalent, namely, by appropriate transformation one can pass from one formulation to the other. This is, however, not so in a stochastic setting where careful distinction needs to be made between "open-loop control" (pre-programmed control) and "feedback control" (control based on past history of the observations).

This report is concerned with fundamental aspects of filtering theory and stochastic control. The main themes of this research have been described in the

comprehensive proposal submitted to the Air Force Office of Scientific Research in September 1981. One of the new directions proposed was to develop a Scattering and Inverse Scattering Framework for estimation problems for random fields. Considerable progress has been made in the direction during the current period of the grant.

It should also be mentioned that the work proposed here is potentially of great benefit to the U.S. Air Force. Increasingly, it is being recognized that ad hoc techniques using linearization and perturbation methods are unsatisfactory and nonlinear theory is ripe for applications. The Kalman filter has played an important role in guidance and control of aerospace vehicles. However, the extended Kalman filter which is used to handle nonlinear situations is not understood from a scientific point of view and often given rise to incurable convergence difficulties. We have made some progress towards alleviating this situation. In particular, we have greatly enhanced understanding about the derivation and functioning of the Extended Kalman Filter.

The control of future aircrafts, large space structures, and aerospace vehicles is a problem of continuing importance from the point of view of designing adaptive systems that operate reliably over

a wide operating envelope. Similarly, the control of advanced jet engines, whose dynamic characteristics change rapidly with operating conditions, pose difficult problems if one wishes to design a control system which accomplishes commanded thrust level changes rapidly, while maintaining fan and compressor stability margins. It would appear that an adequate theory of stochastic control will be essential to solve these problems. Due to the tremendous increase in computing power available and decrease in costs in memory size the problem of dealing with non-linearities is no longer the insurmountable obstacle it was.

2. Research Progress

3.1 Nonlinear Filtering

A long-standing open problem in filtering theory has been to obtain a derivation of the Extended Kalman Filter and explain its qualitative behavior. The Extended Kalman Filter is widely used in aerospace systems and is known to function very well in many situations, but is also known to exhibit divergence phenomenon in the presence of modelling errors. In the paper [1], first steps towards a rigorous derivation of Extended Kalman Filter as well as explaining its qualitative properties were taken. Further work on this problem has been done, and a detailed paper jointly with Wendell Fleming of Brown University is now in preparation. To obtain this result, we use the

stochastic control interpretation of nonlinear filtering described and in the joint paper with Wendell Fleming [2], the Morse Lemma with parameters [3], and the work of Malliavin on Stochastic Jacobi fields [4].

It was suggested that the Stochastic Control Interpretation of Nonlinear Filtering would provide the means for obtaining bounds for nonlinear filtering. A first step in this direction has been taken in the paper mentioned above. Specifically, the analog of the Fisher Information Matrix has been defined. This paper also shows how the nonlinear filtering problem is related to the identification problem. We are now considering examples related to the phase-lock loop to understand better lower bounds on performance that can be obtained using these ideas.

An important open problem for the last few years has been the construction of robust filters using pathwise nonlinear filtering for observations which are unbounded. This problem has been settled in the negative by Sussmann based on preliminary work by Mitter. More specifically, we have shown by example that the normalization constant becomes infinity for certain "physical" observation paths. This suggests that what is needed, is, constructing the normalized conditioned density by examining the robust versions of the numerator and denominator of the Kallianpur-Striebel Formula after "cut-offs" have been introduced. One then has to remove the cut-off in an approriate way. This is a familiar procedure in Statistical Mechanics

but some key monotonicity properties seem to be absent in the problem under investigation.

For further progress on the qualitative behavior on nonlinear filters, it is necessary to understand the small-time and large-time behaviors of these filters. Mathematically, this problem is related to the small-time behavior of conditional diffusions and to large deviation theory. For diffusion processes there now exists a large body of theory developed by Donsker-Varadhan and Ventcel-Freidlin (cf. Bismut [loc. cit] for a review of these ideas). What is needed is a generalization of these ideas to conditional diffusion processes, and we have begun investigations on this problem.

3.2 Linear Estimation, Scattering Theory and Inverse Scattering Theory

We have done research on

(i) The relationship between Estimation Theory and Scattering Theory with a view to obtaining a better understanding of filter synthesis and approximation theory for linear filters and

(ii) The relationship between Estimation Theory and Inverse Scattering Theory with a view to understanding estimation for random fields.

3.2.1 Filtering Theory and Scattering Theory

We have made considerable progress in illuminating the relationship between the work of Adamjan-Arov and

Krein on Scattering Theory and the Theory of Hankel Operators. This work has important consequences for the following problems of stochastic systems theory:

a) Stochastic Realization Theory

b) Approximation of Stochastic Systems

c) Approximation of Filters

Moreover, the concept of a scattering function (in the sense of Adamjan-Arov and Krein) provides a unifying framework for the study of deterministic and stochastic linear systems, both in finite and infinite dimensions. The details of this work will be available in the doctoral dissertation of Y. Avniel and in forthcoming papers by Mitter and Avniel. A preliminary report will be presented in the AMS Summer Course on Linear Algebra and Systems Theory in July 1984.

3.2.2 Estimation Theory and Inverse Scattering Theory

Over the past year, our research has focused on the study of the relations existing between linear estimation theory and inverse scattering theory, and on the development of new inverse scattering techniques of a differential type. We have also studied the application of these techniques to inverse problems such as the inverse seismic or inverse resistivity problem of geophysics, or such as the problem of reconstructing a minimum-phase filter from its phase or magnitude. The results of this investigation are reported in [1]-[9], and will only be reviewed briefly here.

In the area of linear estimation and its relation to inverse scattering theory, it was shown in [1]-[2] that the problem of estimating a one or two-dimensional isotropic random field from noisy observations can be formulated as an inverse scattering problem, where given the spectral density $r(\lambda)$ of the observations $y(.)$, the objective is to construct a second-order differential operator whose spectral density is also $r(\lambda)$. The differential operators that we have considered are the Schrödinger operator $\underline{\Delta} = -\frac{d^2}{dx^2} + V(x)$ and the operator

$$\underline{W} = \begin{pmatrix} -\frac{d}{dx} & r(x) \\ -r(x) & \frac{d}{dx} \end{pmatrix}$$

which is associated to two-component wave systems of the type appearing in the analysis of transmission lines. Then, we showed that the problem of reconstructing $\underline{\Delta}$ or $\underline{W}$ from the spectral function $r(\lambda)$ is identical to the original random fields estimation problem. By exploiting the structure of the Gelfand-Levitan or Marchenko equations which appear in the reconstruction process, we were able to obtain some efficient algorithms for computing the optimum estimation filter. These algorithms generalize the so-called Levinson recursions which in one dimension solve the prediction problem over a finite interval. The fast

algorithm that we have obtained for estimating isotropic two-dimensional random fields is the first to be derived in dimension higher than one. We have been able recently to use this algorithm to construct maximum entropy spectral estimator of two-dimensional (2-D) random fields. This method for constructing 2-D maximum entropy spectral estimator is considerably more efficient than existing techniques, which had to rely on iterative techniques.

In addition, we are also currently studying the implications of our estimation results and of their links with inverse scattering theory for the design of optimum estimation filters with lattice architectures which reflect the two-component wave model which was selected. The advantage of lattice architectures is that they are modular, stable, and have been shown to be highly insensitive to noise. The existence of such filter implementations in one dimension is known, but our work also applies to higher dimensions.

In the area of inverse scattering theory, in [3], [6], we have obtained a new class of differential inverse scattering methods which operate on a layer stripping principle and reconstruct a scattering medium recursively, layer by layer. These inverse scattering methods generalize an algorithm introduced by Schur in 1917 for testing the boundedness of a function which is analytic inside the unit circle. These recursions also arise when we want to factor a

symmetric Toeplitz operator in causal times anticausal form, or when we want to compute the Cholesky factorization of a symmetric Toeplitz matrix. Because of their recursive nature, these algorithms are easier to implement than traditional inverse scattering methods (introduced by Gelfand and Levitan, Kay and Moses, Krein, Marchenko, Faddeev...) which rely on the solution of integral equations. The differential inverse scattering methods that we propose have been related to these integral equations in [3], where an extremely simple derivation of all integral methods which is based on causality is given.

An overview of the applications of differential inverse scattering techniques to transmission line analysis, inverse problems of geophysics, linear estimation theory and even the reconstruction of lossy systems is given in [6]. In [4] the Schur algorithm was used to solve the inverse seismic problem for a one-dimensional layered medium described by acoustic equations. By probing the medium <u>obliquely</u> with plane waves at two different angles, and by running two distinct sets of recursions corresponding to these two experiments, it was shown that <u>both</u> the density $\rho(x)$ and velocity $c(x)$ of the medium can be reconstructed as functions of depth. The point source case (i.e. the case when the probing wave is a spherical wave) was also considered, and it was shown that the data can be transformed into the data for oblique incidence

experiments by slant-stacking, i.e. by performing Radon transforms. These results were generalized in [5] to the case when the medium which is probed is described by the equations of elasticity, in which case the problem becomes harder since both pressure (P) and stress (S) waves coexist and can be converted into one another. By generalizing the layer-stripping approach of [3], [6] to this system (which is a differential system of order 4) we were able to reconstruct the density $\rho(x)$ and the Lamé parameters $\lambda(x)$ and $\mu(x)$ as functions of depth.

Preliminary numerical results which apply these reconstruction techniques to synthetic data seem promising. This was somewhat expected since the Schur algorithm is known to be numerically stable. We are presently studying the impact of noise on our reconstruction techniques, as well as developing several methods which rely on taking more data and using least-squares methods to reduce the influence of noise. In addition, since our results hold when the medium is probed by plane waves at angles beyond the critical angle of incidence, we are studying the impact of turning points on our reconstruction methods, and more specifically whether the waves <u>transmitted</u> back to the surface beyond the turning point can be used in our reconstruction procedure, in addition to the reflected (scattered) waves which we are presently using. A full account of all these results will be given in [9].

Recently, we have also considered inverse scattering problems for which the reflection coefficient is rational. Several reconstruction methods for this problem were developed in the past, but these methods are relatively inefficient since they rely on solving systems of linear equations, or computing determinants for every reconstructed value. Instead, by using the analogy with linear estimation theory, in [7] we were able to obtain a reconstruction procedure which relies on a state-space model of the reflection coefficient, and which is similar to the Kalman filter of linear filtering theory. The advantage of this procedure is that it is recursive, and requires fewer operations than existing methods.

The problems mentioned above are not the only ones for which layer-stripping ideas can be used. In [6], it was shown that the Schur algorithm could be generalized to deal with the case when the medium that we want to reconstruct is lossy. In this case, it is necessary to use both transmission and reflection data, and two coupled Schur algorithms need to be used. This procedure was illustrated for the reconstruction of lossy transmission lines. In [8], we studied the inverse resistivity problem of the earth, which arises when the earth is probed by injecting some direct current, and when the potential on the surface of the earth is measured. The main feature of this problem is that it is not an inverse scattering

problem since the equation for the earth 's potential in <u>elliptic</u>. Nevertheless, a mapping procedure can be used to transform this problem into an equivalent inverse scattering problem. This mapping procedure can be interpreted in terms of Maxwell's method of images which models the inhomogeneities of the earth's resistivity by the introduction of fictitious current sources inside the earth. We are also investigating the application of this method to the study of the inverse seismic problem where the earth is probed obliquely by plane waves at <u>two fixed frequencies</u>, and for all lateral wavenumbers. The results that we have obtained will be described in [9].

Finally, it should be mentioned that 1-D inverse problems are quite simple when compared to real world inverse problems which are usually three-dimensional. We are now starting some work on 3-D inverse problems of geophysics, which relies on combining wave migration techniques with Born, or WKBJ approximation techniques of inverse scattering theory.

4. <u>New Research Initiated during this Grant Period</u>

We have begun research on image processing and image understanding using the theory of Markov Random Fields. The objective is to develop a theory of stochastic geometry which will be useful in recognition of geometric shapes present in a noisy image.

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Section 3.2

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L. "Stochastic Variational Problems," Berkeley-Ames Conference on Nonlinear Problems in Control and Fluid Dynamics, Berkeley, California, May 31-June 10, 1983.

2. Lectures on Pathwise Nonlinear Filtering, Scuola Normale Superiore, Pisa, Italy, June 1983.

3. "Une Contre-example dans le Filtrage Trajectorielle," Institute National de Recherche en Informatique et Automatique, Sofia-Antipolis, France, June 27, 1983.

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5. "A Counterexample in Pathwise Nonlinear Filtering," Italian National Meeting on Filtering and Stochastic Control, Padova, Italy, September 12-15, 1983.

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